Graduate Program in Economics
Universitat Autònoma de Barcelona

Optimization.
Final Exam
December 2, 2015.

1. Determine the value of the parameter a so that the function

$$f(x, y, z) = ax^2 + y^2 + 2z^2 - 4axy + 2yz$$

is convex.

2. Consider a profit maximizing firm using two inputs x, y to produce an output using the following production function: $f(x, y) = 2x^2 + y^2$. Suppose $p > 0$ is the price of the output, and w_x and w_y are the prices of the inputs. Suppose $w_x = w_y = w$. Suppose also that the available inputs are $x + y = 1$.

   a) Verify that in the optimum the gradient of the objective function π is aligned with the gradient of the restriction g i.e., $\nabla\pi(x^*, y^*) = \lambda^*\nabla g(x^*, y^*)$

   b) Use λ^* to asses the impact on $\pi(x^*, y^*)$ if the restriction is relaxed by 10%.

3. Solve

$$\max_{x_1,x_2} x_1 + \ln x_2 \quad \text{s.t.} \quad \begin{cases} x_1p_1 + x_2p_2 \leq w \\ x_1 \geq 0 \\ x_2 > 0 \end{cases}$$

with $p_1, p_2, w > 0$.

4. Solve the following dynamic problem

$$\max_{\{c_t\}} \sum_{t=0}^{2} \left(1 + x_t - c_t^2\right) + x_3 \quad \text{s.t.} \quad \begin{cases} x_{t+1} = x_t + c_t, \ t = 0, 1, 2 \\ c_t \geq 0 \\ x_0 = 0 \end{cases}$$

Assume for simplicity $\beta = 1$.